Invention under uncertainty and the threat of ex post entry

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Abstract

This paper proposes a theoretical framework for studying the invention of new products when demand is uncertain. In this framework, under general conditions, the threat of ex post entry by a competitor can deter invention ex ante. Asymmetric market power in the ex post market exacerbates the problem. The implications of these general results are examined in a series of examples that represent important markets in the computer industry. The first is a model that shows how an operating system monopolist, by its mere presence, can deter the invention of complements, to its own detriment as well as that of society. The implications of policies such as patent protection, price regulation, and mandatory divestiture are considered. Three additional examples consider the ability of a monopolist in one market to commit to bundling an unrelated product, a pair of horizontally differentiated firms that can add a new feature to their products, and a platform leader that can be challenged in its base market by the supplier of a complementary product. **JEL Classifications**: L12, L13, O31

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1 Introduction

Consider two firms, each interested in developing the same new product. Suppose that demand for the product is known, and the firms decide simultaneously whether or not to enter the market.[1] If monopoly profits are high, duopoly profits are low, and development costs are moderate, then there are two pure strategy equilibria, in each of which exactly one firm enters. If after the first firm enters, the firm that chose not to enter initially is given a second chance to enter, it will decline the opportunity.

This simple picture changes dramatically when demand for the new product is unknown. Demand uncertainty can lead both firms to forego entry even when development costs are moderate, leaving a promising market untapped. In Section 3 I develop a two-period framework for studying this problem, and show that invention is foregone when a firm with moderately high entry costs can benefit ex post from free riding on the demand information revealed by the first mover. Because the first mover cannot internalize this informational externality, some invention that may be both privately and socially beneficial is foregone due to the threat of ex post entry.

More precisely, if the first mover has entered the market and revealed high demand, a second mover will also enter the market ex post if its prospective profits as a duopolist outweigh its entry costs. Ex ante, the first mover recognizes it will earn either low monopoly profits if demand is low, or low duopoly profits if demand is high. Hence the first mover will not enter the market ex ante unless its entry costs are very low. As for the second mover, why does it not elect to become the first mover? Because demand is uncertain, its expected profits can be too low to justify incurring the entry costs, even if it would enter ex post if demand were known to be high. Though this reasoning is asymmetric, it can apply to symmetric situations, where the possibility that each firm could play the ex post role of the second mover deters the other from entering ex ante in its role as the first mover.

I apply these general results to four examples drawn from stylized scenarios in the computer software industry. Section 4 considers a base system monopolist that also competes in a complementary application market. The monopolist owns a selling advantage since consumers must purchase a base system in order to use the application. In this setting, the base system monopolist, by its mere presence, can depress the invention of complements—to its own detriment. In Section 5, I adapt the basic framework to three other examples of competition common in software markets: the bundling of software applications; the addition of new features to horizontally differentiated products; and an example inspired by the "divided technical leadership" hypothesis of Bresnahan and Greenstein (1999), in which the salient threat to a platform leader is a complementor rather than a new entrant.

[1] In this paper I use "development", "investment", and "entry" as synonyms; "invention" and "innovation" refer to the first act of investment in a particular product category.

Since the foregone invention result predicts the non-existence of certain products, the question arises whether the framework's predictions are falsifiable. However, the steadily falling development costs that characterize computer software technology imply that the development costs for any given software application will eventually leave the region of foregone invention. So inventions that are foregone in one period are likely to be observed in later periods. Historical research into firms' internal records and communications may be able to identify cases in which development efforts were abandoned or delayed due to a threat of ex post entry. Similarly, a natural experiment, such as a regime change in antitrust or patent policy, a change in the monopoly status of a complementor, or a merger of competitors, may cause a shift in development activity that could be traced to a change in whether invention is foregone. Observations like these may help distinguish foregone invention from alternative explanations, such as the possibility that firms are merely pessimistic about demand.

The main result—that invention may be foregone due to the threat of ex post entry—depends on two key forces. The first is the uncertainty inherent in the process of invention, which puts a wedge between ex ante and ex post incentives. The second is entry cost heterogeneity across firms, which implies that some firms may have an incentive to enter an empty product market while other firms will not enter until the level of demand has been revealed. A third force, asymmetry in the ex post selling stage, although it is not necessary for the result (as the example in Section 5.2 demonstrates), amplifies the problem because a high entry cost firm with a selling advantage has a greater incentive to free ride on the information externality that is generated when a low entry cost firm reveals demand.

In software markets, uncertainty takes two primary forms: technological uncertainty and demand uncertainty. Both stem from the inherent complexity of computer software—the amount of effort and coordination among programmers required to write a software program is unknown, and potential consumers for a product that does not exist may have difficulty perceiving or articulating their needs for the product.[2] In this paper all the uncertainty is on the demand side, but a similar result would obtain if demand were known but the technological uncertainty faced by an early entrant were more severe than the uncertainty faced by subsequent entrants.

Cost heterogeneity across firms is central to many of the results. In the model of Section 4, for example, if entry costs were identical across firms then there would be no reason for a base system monopolist to ever decline to enter a promising complementary application market, and consequently there would be no reason for an independent firm ever to contemplate entry. Firms might face substantially different entry costs for any of a host of reasons. There might be legal constraints, such as patents or antitrust concerns; there

[2]Even after purchasing a software product, they may engage in what Bresnahan and Greenstein (1996) term "co-invention": the process of discovering how the software's capabilities interact with existing organizations and habits, and of applying the software to uses not foreseen by its inventors.

might be history-dependent effects, such as learning-by-doing or unique human capital retained by a particular firm; there might be consequences of past decisions regarding internal organization, financing, or employee compensation that are costly to change. Perhaps most importantly, firms of different sizes face different problems of providing their employees with incentives. However, invention can be foregone in contexts without cost heterogeneity, as in the model of Section 5.2.

Asymmetry in the ex post selling stage can arise due to complementarity: a firm with a monopoly over an existing product can enter a complementary market with a competitively low price, yet still profit by the accompanying increasing in demand for its monopoly product. Asymmetry can also arise as a consequence of substitutability: if a newly introduced product is complementary to a substitute for a firm's existing product, the firm may defend itself by developing a competing product (Sections 5.2–5.3 give examples).

2 Related literature

The problem of foregone invention is fundamentally one of appropriability—when a firm cannot guarantee itself a monopoly over its invention, its incentives to invent are reduced. To solve the appropriability problem, governments commonly employ patents to guarantee a monopoly to each inventor. But patents can be harmful when invention would occur without them, because they reduce ex post competition. Such competition is particularly important when there is scope for incremental quality improvement, because a firm that faces no competition ex post has less incentive to improve on its invention.[3] Fine tuning the patent system to avoid such problems would require unrealistic capabilities on the part of the government. Finally, even if patents could be helpful in concept, it is difficult to apply the legal framework of the patent system to categories of software applications, because software applications typically make use of previously patented technologies.[4]

One branch of the literature on innovation focuses on patent races, where several firms each have the opportunity to invest in developing a product for which demand is known, but the success or failure of their development efforts is dependent on their investments.[5] Adding demand uncertainty to such models adds no new insights, since the prize for winning the race is simply the expected monopoly profits.[6]

[3] Bessen and Maskin (2002) find that patents can prevent inventors from looking forward to imitating subsequent improvements invented by others.

[4] See Bessen (2005). Furthermore, Anton and Yao (2004) argue that a broadly based patent law does not necessarily threaten penalties severe enough to prevent imitation in every circumstance.

[5] Loury (1979), Dasgupta and Stiglitz (1980), and Reinganum (1981, 1982) are early works in this literature; recent contributions include d'Aspremont, Bhattacharya, and Gérard-Varet (2000), Denicolò (2000), Weeds (2002), and Doraszelski (2003).

[6] Weeds (2002) models demand as a stochastic process, leading firms to want to delay innovation in order to see if demand is likely to be high. However, no new information is revealed by the actions of the firms.

Another branch focuses on optimal patent policy given the tradeoffs between incentives for the initial invention and the benefits of subsequent competition or sequential innovations.[7] But these kinds of models give one firm an exogenous monopoly over the ability to invent the primary innovation; the question arises why other firms cannot also develop it. My basic framework can address this question by varying the ability of each firm to invest in the primary innovation, i.e., by parameterizing development costs. The key insight is that under these circumstances imitation poses no threat to innovation unless there is uncertainty over the demand for the invention (or, as in Section 5.3, strategic complementarity).

A different branch of the innovation literature is more closely related. Jensen (1992a, 2001, 2004) examines a dynamic environment with two firms that can invest in an innovation of uncertain success, as in the present paper.[8] Though the identity and behavior of the first entrant are their main concerns, these papers also identify the threshold probability of success below which neither firm invests. My results generalize the foregone invention content of these analyses by allowing the cost of entry to vary across firms, characterizing the region of foregone invention as a function of the cost parameters.

Asymmetric market power has received significant attention in the industrial organization literature. When it is not the simple consequence of an entry barrier or demand advantage, asymmetric market power typically stems from some sort of complementarity. It is well established that complementarity between markets makes a vertically integrated monopoly the socially preferred market structure from a static perspective.[9] Recent work has investigated some dynamic consequences of these static results. For instance, a firm with products in multiple markets can tie its products together to deter entry, induce exit, or extend its monopoly to new markets, to the detriment of society.[10] In the context of innovation, Stefanadis (1997), Farrell and Katz (2000), Choi and Stefanadis (2001), and Heeb (2003) show that, compared to a level playing field, incremental quality improvement or cost reduction is relatively more attractive for a complementary monopolist and relatively less attractive for other firms. Unlike my basic framework, these models do not allow firms to take advantage of information revealed by their competitors.

[7]Notable examples include Gilbert and Shapiro (1990), Klemperer (1990), Gallini (1992), Chang (1995), Green and Scotchmer (1995), Schankerman and Scotchmer (2001), and Anton and Yao (2003, 2004). Also related is Milgrom and Roberts (1982) and the subsequent literature on limit pricing and signaling, which considers an incumbent who can signal market conditions to a potential entrant. A separating equilibrium in this signaling game can be seen as the second stage of the basic framework considered here.

[8]In addition, Jensen (2003) investigates a model in which firms act in a fixed sequence and demand is revealed gradually. Benveniste, Busaba, and Wilhelm (2002) consider a related problem concerning the revelation of market conditions, but in an environment without ex post competition. Also, Reinganum (1983) and Jensen (1992b) consider one-period, two-firm problems with invention under uncertainty.

[9]See Chen and Ross (1998), Economides and Salop (1992), and the seminal work of Cournot (1838).

[10]See Whinston (1990), Bakos and Brynjolfsson (2000), Nalebuff (2000, 2004), and Carlton and Waldman (2002).

3 The basic framework

The framework consists of a two-period game, $t = 1, 2$, between two firms. There is an empty potential market with unknown demand, and either firm $j \in \{M, X\}$ (M for "monopolist," X for "alternative") may enter the market after expending an idiosyncratic entry cost $c_j \geq 0$. Firm j's entry decision in period t is written as in_j^t or out_j^t. Demand in the market is revealed to both firms once any firm enters.[11] Entry decisions are made simultaneously at the start of each period; exit is not considered and entry costs cannot be recovered.

In the basic framework, per-period profits are the primitive element.[12] (The examples in later sections model pricing decisions in detail.) Let ω be the level of demand, which is initially unknown but whose probability distribution is common knowledge. The payoff for firm j in period t given demand ω is indicated by $\pi_j^t(s, \omega)$, where $s \in \{\emptyset, M, X, MX\}$ (for short) is the *market structure*, or the set of firms that have already entered. In the absence of entry ($s = \emptyset$), ω is irrelevant, so the payoff for firm j is written simply as $\pi_j^t(\emptyset)$. All these details—including c_M and c_X—are common knowledge.

The extensive form of this game is shown in Figure 1. The probabilistic nature of ω is indicated by the choice of "Nature," which chooses at the nodes marked "N". Note that if neither firm enters in the first period, then neither of them is allowed to enter in the second period. I make this restriction to focus on the incentives for ex ante entry when there is a possibility of ex post entry.[13]

To model realistic situations it helps to impose some assumptions on the structure of profits. The following assumptions are maintained for the remainder of this section and are satisfied by all the examples in later sections.

Assumption 1 (Relevance). $\mathbb{E}\left[\pi_j^1(j, \omega) + \pi_j^2(j, \omega)\right] > \pi_j^1(\emptyset) + \pi_j^2(\emptyset)$ *for all* j.

Assumption 2 (Common demand). $\omega \in [0, 1]$ *and* $\pi_j^t(s, \omega)$ *is increasing in* ω *for all* $j \in s$.

Assumption 3 (Competitiveness). $\pi_j^t(j, \omega) > \pi_j^t(MX, \omega)$ *for all* j*, all* t*, and all* $\omega > 0$.

Assumption 4 (Positive profits). $\pi_j^t(MX, \omega) > \pi_j^t(-j, \omega)$ *for all* j*, all* t*, and all* $\omega > 0$.

These assumptions define a class of interesting situations. If *relevance* holds, then the empty market offers enough profit potential that each firm would enter if its entry cost were

[11] It is not necessary for the results that demand be revealed ex post; it is enough that entry by the first firm reveals some information to the second firm. Indeed, if each firm could privately observe a noisy signal of demand before period 1, then entry by the first firm would reveal that it had observed a signal of high demand. The second firm would then update its estimate of demand. Invention is foregone if the first firm fails to enter in fear of the second firm entering after it updates its estimate.

[12] Differences in profit functions across time do not play an important role in the basic framework. However, expressing the profits as a function of time aids in mapping the framework onto applications.

[13] This restriction also allows the two-period framework to be mapped onto infinite horizon models, as in the later sections, in which each firm may enter at any time. In such models the incentives to enter are stationary; i.e., if invention is ever rational in any period, then it is rational in the first period.

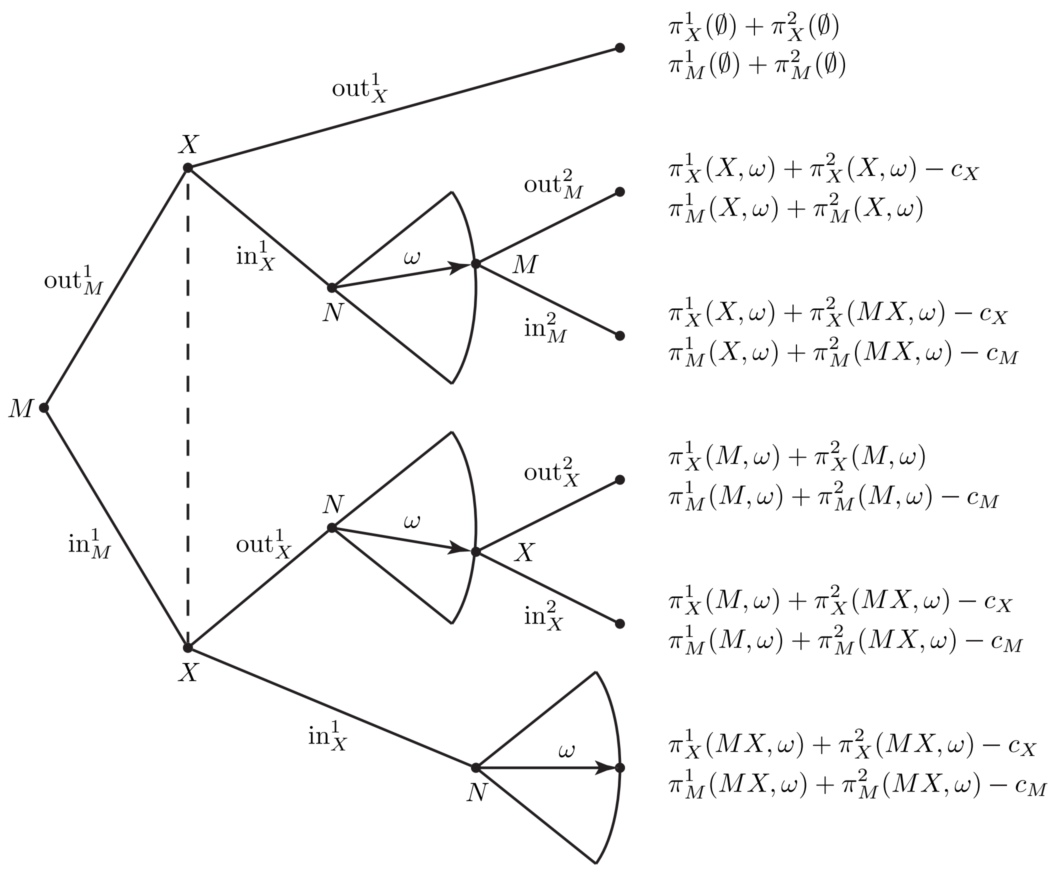

Figure 1: Extensive form of the basic framework

sufficiently low and it knew that it would gain a monopoly. If *common demand* holds, then demand is characterized by a single parameter and an increase in that parameter leads to higher profits for every firm in the market. If *competitiveness* holds, then each firm prefers to monopolize the market rather than share it with a competitor. *Positive profits* means that revenues for any firm in the market will always cover its operating costs, so that the act of entry risks at most the entry costs.

An additional definition formalizes the idea that a firm faces a disadvantage in the ex post market.

Definition 1. Investments are **strategic substitutes** for firm j if, for all t,

$$\mathbb{E}\big[\pi^t_j(MX,\omega) - \pi^t_j(-j,\omega)\big] < \mathbb{E}\big[\pi^t_j(j,\omega)\big] - \pi^t_j(\emptyset). \tag{1}$$

That is, Firm X (for example) gains less from investing when Firm M also invests than it gains from investing when Firm M does not invest. If Firm X's profits when it does not enter the market do not depend on Firm M's action (i.e., $\pi_X^t(M,\omega) = \pi_X^t(\emptyset)$ for all ω and all t), then competitiveness implies that investments are strategic substitutes for Firm X. Otherwise, that investments are strategic substitutes for Firm X implies that Firm M's entry does not depress Firm X's profits (e.g., in some related market) too much when Firm X does not invest. Investments can be strategic substitutes for both firms, but in the results that follow they need only be strategic substitutes for at least one firm—which I name Firm X.

When Firm X perceives investments as strategic substitutes, there are situations in which Firm X does not enter a promising market. Define $\hat{c}_j$ as the entry cost at which firm j is exactly indifferent between investing and not investing in the absence of firm $-j$:

$$\hat{c}_j \equiv \mathbb{E}\big[\pi_j^1(j,\omega) + \pi_j^2(j,\omega)\big] - \pi_j^1(\emptyset) - \pi_j^2(\emptyset). \tag{2}$$

If Firm M were for some reason unable to enter the market, then at entry cost $\hat{c}_X$ Firm X would be indifferent between entering and not entering. Since Firm M is able to enter ex post, there exists a threshold entry cost $\tilde{c}_M > 0$ below which it will indeed enter with positive probability once demand is revealed. By competitiveness, when $c_M < \tilde{c}_M$ Firm X's strict best response given any entry cost in a neighborhood around $\hat{c}_X$ is not to enter, even though it would strictly prefer to enter when $c_M > \tilde{c}_M$. Lemma 2, in the Appendix, makes this argument formally.

Combining the results for the individual firms reveals the conditions under which neither firm enters a promising market. It is helpful to visualize each firm's optimal first period entry decision as a function of its entry cost profile, as is shown in Figure 2A for Firm X. The region in which Firm X does not enter the market solely due to the possibility of entry by Firm M covers intermediate values of c_X and low values of c_M. Overlaying the regions in which Firm M and Firm X do not enter in the first period, as is shown in Figure 2B, reveals the region of "foregone invention":

Definition 2. The **region of foregone invention** is the maximal set of cost vectors $\{(c_2, c_2)\} \subset \mathbb{R}^2$ for which (i) no firm enters in any subgame perfect equilibrium, (ii) but at least one firm would enter in a subgame perfect equilibrium if it were assured a monopoly.

The region of foregone invention covers an area of moderate entry costs, because very high entry costs or very low entry costs make the deterrence of invention either difficult or moot. Specifically, if Firm X has very low entry costs it is not likely to be deterred by the prospect of Firm M's entry, while if Firm M has very high entry costs it is less likely to enter and thus poses less of a threat to Firm X. Similarly, if Firm X has very high entry costs then it is unlikely to enter even without the threat of Firm M's subsequent entry, while if Firm M has low entry costs then it is likely to enter ex ante regardless of whether it deters Firm X. It is quite possible that these effects that operate against foregone invention may

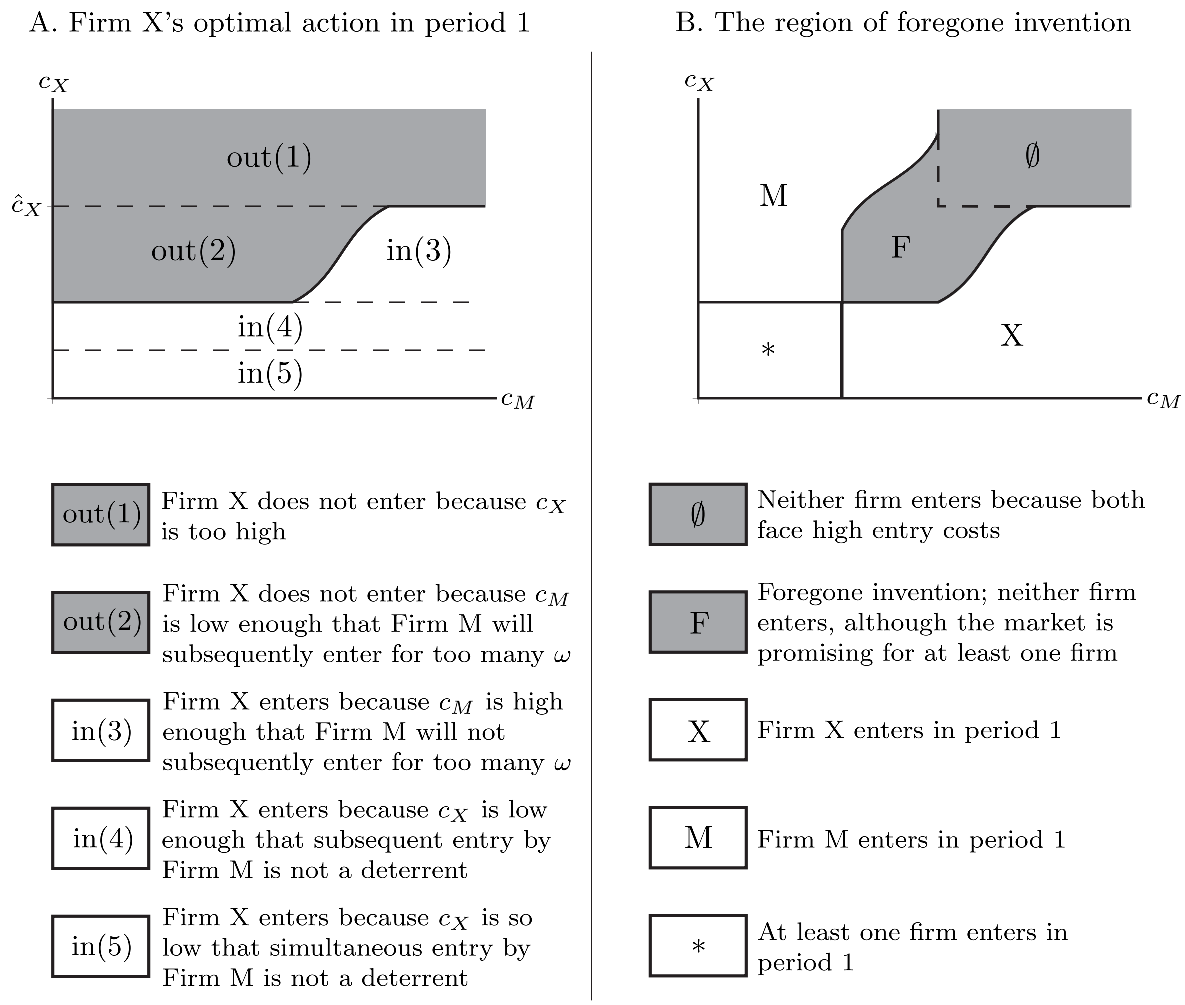

Figure 2: The region of foregone invention in the basic framework Each graph displays the cost space, with c_M on the horizontal axis and c_X on the vertical axis.

overlap, in which case the region of foregone invention is empty. Figure 2B suggests that the region of foregone invention has positive measure if the curves that divide entry from non-entry for each firm do not intersect at $(\hat{c}_M, \hat{c}_X)$. That this is indeed the case is implied by the following proposition.

Proposition 1. *Let $s_j^*(\omega, c_j)$ indicate the market structure that results from Firm j's sequentially optimal choice after Firm $-j$ enters and reveals ω:*

$$s_j^*(\omega, c_j) = \begin{cases} MX & \text{if } \pi_j^2(MX, \omega) - c_j > \pi_j^2(-j, \omega) \\ -j & \text{otherwise.} \end{cases} \quad (3)$$

In the basic framework, if investments are strategic substitutes for Firm X and

$$\mathbb{E}\big[\pi_X^2(X,\omega)\big] > \mathbb{E}\big[\pi_X^2\big(s_M^*(\omega,\hat{c}_M),\omega\big)\big], \tag{4}$$

then the region of foregone invention has positive measure.

Eq. 4 requires that Firm M has an ex post incentive to enter when its entry cost is high enough that it does not enter ex ante. The proof (in the Appendix) uses Assumptions 1–4 and Eq. 4 to establish that $(\text{out}_M^1, \text{out}_X^1)$ is an equilibrium at entry cost profiles near $(\hat{c}_M, \hat{c}_X)$, while Firm X's perception that investments are strategic substitutes rules out the possibility that $(\text{in}_M^1, \text{in}_X^1)$ is also an equilibrium.[14]

Under what conditions is Eq. 4 likely to be satisfied? The left hand side is high when Firm X's monopoly profits in the new product market are likely to be high; the right hand side is low when Firm X's duopoly profits are likely to be low and when Firm M is likely to enter ex post. Thus the condition is more likely to be satisfied if Firm M has an advantage over Firm X in the ex post market, since Firm M is more likely to enter ex post if its duopoly profits are likely to be high. Although there are symmetric conditions that can lead to foregone invention, symmetry increases or decreases both firms' duopoly profits in tandem, leading to counteracting effects on the right hand side of Eq. 4. But as Firm M grows stronger ex post at the expense of Firm X, ex post entry becomes more likely and Firm X's expected profits fall unambiguously. Hence foregone invention is a more serious problem when ex post market power is asymmetric. In subsequent sections I explore several examples of asymmetric market power, in which the asymmetry derives from one firm's market power in a related market.[15]

Note that the basic framework does not allow the first entrant to erect entry barriers, since $\pi_j^2(MX,\omega)$ does not depend on whether Firm M or Firm X entered first. The framework could be expanded to allow duopoly profits to vary with the identity of the first entrant. As profits for the second entrant fell, the region of foregone invention would shrink and eventually disappear. Just like patents, sufficiently strong entry barriers eliminate the foregone invention problem by shutting down ex post competition.

[14] These are sufficient, but not necessary, conditions for foregone invention. It is possible for invention to be foregone when Eq. 4 does not hold, such as when the curves that divide entry from non-entry for each firm intersect at $(\hat{c}_M, \hat{c}_X)$ but also intersect at other points.

[15] There are other possible sources of asymmetry, such as asymmetric operating costs. For example, in a two period model with Cournot competition the following parameterization yields a region of foregone invention with positive measure: demand $\omega(10-p)$, uncertainty $\omega = 1$ with probability γ and $\omega = 0$ otherwise, and marginal costs of 0 for Firm M and 3 for Firm X.

4 Invention to complement a base system

This section shows that a computing platform with a base system monopolist is liable to experience foregone invention. The problem is that the base system monopolist possesses an ex post advantage in markets for complementary products since it can enter a complementary market with a low price yet still benefit from the increased demand for its base system. The asymmetric complementarity between the two markets amplifies the problem of foregone invention, to the potential detriment of both firms as well as to social welfare.

Although designed to represent aspects of the computer software industry, the model applies to any set of markets in which a monopolist's "base system" is required for the operation of other "applications" and in which there is potential for the invention of new applications. Gawer and Henderson (2004) and Gawer and Cusumano (2002) pose an example in the computer hardware industry: Intel, given its dominant position in the market for microprocessors, has an incentive to enter complementary markets because it can internalize the complementarity. But Intel also benefits when other firms innovate in these areas, because such innovations increase the market for Intel's core products. When both Intel and another firm are aware of an opportunity to open a new market, foregone invention may be a problem if either demand in this market is unknown (as in the model), or, equivalently, if the technological viability of the product is unknown. Intel has developed organizational structures that may help it commit not to "trample all over everybody's business" (Gawer and Henderson), and it has kept most of its efforts focused on its core business. However, Intel has entered a number of already-occupied complementary markets.[16] This section gives a theoretical rationale for the contrast between Intel's stated concern for complementors and its apparent inability to fully commit not to behave aggressively toward them.

Naturally, the model can also address Microsoft's dominance over computer operating systems and the many applications that can run on top of Windows operating systems. Microsoft's success in markets for Windows applications—internet browsers, e-mail clients, media players, spreadsheets, word processors, etc.—illustrates the strength of its ex post advantage. Accordingly, the results in this section suggest that a market structure with a base system monopolist may pose hazards that retard innovation if, among the various potential products that can work with the base system, there are some for which independent firms have lower development costs than the monopolist.

[16] Including videoconferencing, digital cameras, chipsets, motherboards, home networking, wireless networks, and network adapters.

4.1 The model

Technology and firms Let $\{b, a\}$ be the set of possible products—b for "base system" and a for "application". Firm M holds an exogenous monopoly over the existing base system market. Either firm may enter the application market by incurring a fixed cost $c_j > 0$. The application requires the base system in order to provide value, but the base system can provide value without the application, i.e., the base system is "essential."[17]

Markets and consumers Once firms have had the opportunity to develop the application, they simultaneously set prices, and produce at zero cost any products that they have developed. The set of branded products available for sale to consumers is $s \subset \{b, a\} \times \{M, X\}$, where, for shorthand, $k_j \in s$ is product k sold by firm j at price p_j^k. The set s also represents the "industry structure"—the combination of firms present in the application market.

There is a continuum of consumers with total mass 1, and each consumer i values product k by the amount $\theta_i^k \in [0, 1]$.[18] In each period, each consumer i purchases a basket of branded products $s_i \subset s$ to maximize quasilinear utility: $\sum_{k_j \in s_i} (\theta_i^k - p_j^k)$.[19] Since the base system is essential, if s_i is non-empty then it must contain b_M. Consumer values for the base system, θ_i^b, are distributed uniformly on $[0, 1]$, while, prior to the first period, demand in the application market is unknown: with probability γ, θ_i^a are distributed uniformly on $[0, 1]$; otherwise $\theta_i^a = 0$ for all i.

Dynamics The profits for each firm under each market structure can be expressed as payoff functions in the basic framework. This is accomplished by looking at the problem through the lens of an infinite period model[20] in which the firms share a discount factor $\delta \in (0, 1)$, and letting the second period of the basic framework summarize the discounted profits in periods $2, \ldots, \infty$. Since $s = MX$ is an absorbing state, the equilibrium can still be found by backward induction. As in the basic framework, each period t consists of two stages, an investment stage and a selling stage. Once any firm enters the application market, demand is revealed to all. Though the context is dynamic, I assume that market

[17] Heeb (2003) also models an essential base system. Earlier work tends to consider two products used in fixed proportions; e.g., Economides and Salop (1992), Matutes and Regibeau (1998), Farrell and Katz (2000), and Choi and Stefanadis (2001). However, Firm M's pre-existing base system monopoly is inconsistent with a fixed proportions assumption.

[18] Thus applications a_M and a_X are perfect substitutes. Adding a measure of product differentiation could quantitatively change the problem of foregone invention, but would not alter the qualitative conclusions.

[19] Durable goods considerations in a dynamic oligopoly setting, as well as other similar concerns like network effects, are beyond the scope of this paper. But the results in Section 3 indicate that the phenomenon of foregone invention is general in the set of all models that satisfy the conditions that Proposition 1 places on the profit functions, and this set can accommodate such concerns.

[20] The infinite horizon provides stationary payoffs, whereas a two period model would force payoffs in the second period to be perceived differently from the perspectives of the first period and the second period.

pricing in each period reflects the Nash equilibrium of a one-shot simultaneous pricing game; Section 4.4.1 considers the possibility of collusive pricing.

The state vector consists of the sets of products that have been developed by the firms together with the common prior distribution over demand for the application. The state is summarized (with some abuse of notation) by the pair $\langle s, \omega \rangle$, where $s \in \{\emptyset, M, X, MX\}$, $\omega = 1$ if demand for the application is known to be high, $\omega = 0$ if demand for the application is known to be low, and $\omega = \gamma$ if demand for the application is yet unknown.

4.2 Static prices and profits

This section describes the static market equilibria of the Base System model under various industry structures. The derivations of these static equilibria are straightforward and thus omitted. Consumer choices are shown in Figure 3, where the space of consumer valuations is divided into regions according to which consumers purchase which baskets, with $\theta^b \in [0, 1]$ on the horizontal axis and $\theta^a \in [0, 1]$ on the vertical axis. Observe that whenever Firm M offers the application, assuming that it prices its two products separately is without loss of generality, since offering a bundle would be equivalent to setting a price of zero for the application. I call this "trivial bundling" when it arises in equilibrium.

Base system monopoly When there is no application (i.e., ω is either γ or 0), Firm M faces a simple monopoly problem. It sets its price for the base system at $p_M^b = 0.5$, yielding per-period profits of $\pi_M(\emptyset) = 0.25$. Consumer choices are displayed in Figure 3A.

Integrated monopoly Integrated monopoly arises when Firm M monopolizes both markets (with high demand for the application, i.e., $\omega = 1$). Firm M sets prices $p_M^b = 0.667$ and $p_M^a = 0.167$, resulting in per-period profits of $\pi_M(M, 1) = 0.546$.[21] Consumer choices are displayed in Figure 3B. The application price is low because the application makes the base system more valuable, allowing Firm M to raise the base system price. This illustrates how Firm M internalizes the complementarity, yielding benefits for consumers as well as itself.[22]

Bilateral monopoly A bilateral monopoly arises when Firm M monopolizes the base system market and Firm X monopolizes the application market (with $\omega = 1$). The firms set equilibrium prices of $p_M^b = 0.586$ and $p_X^a = 1 - p_M^b$. The per-period profits are $\pi_M(X, 1) = 0.343$ and $\pi_X(X, 1) = 0.172$. Consumer choices are displayed in Figure 3C. The price for the application is higher than under an integrated monopoly because Firm X cannot reap

[21] For simplicity, I use notation as if three-digit approximations are exact.

[22] Note that the change is not a Pareto improvement, as some consumers who would buy the base system under a base system monopoly are priced out of the market under integrated monopoly.

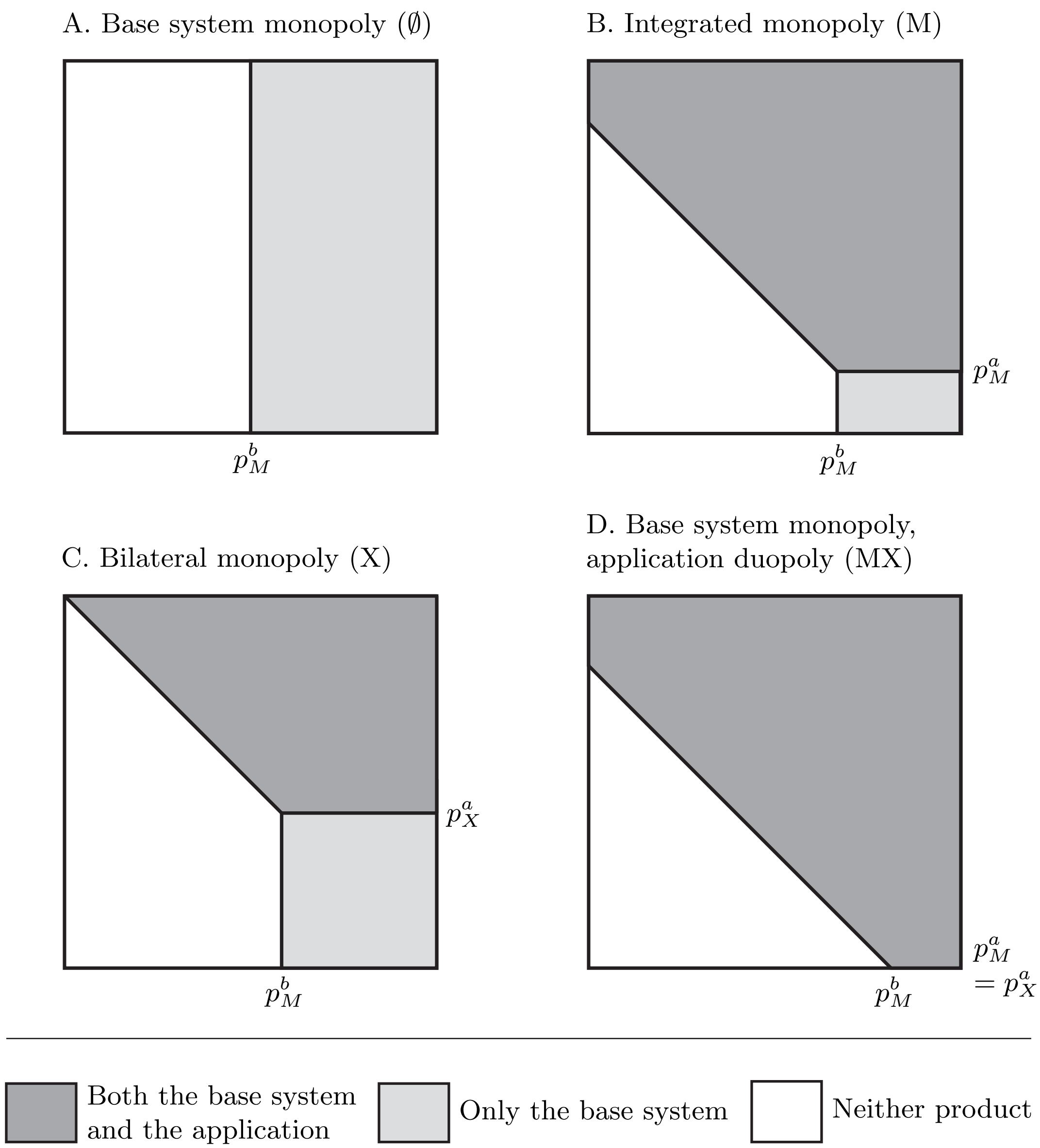

FIGURE 3: CONSUMER PURCHASES AS A FUNCTION OF TASTES IN THE BASE SYSTEM MODEL Each graph displays the space of consumer tastes, with θ^b on the horizontal axis and θ^a on the vertical axis. Regions are shaded according to which consumers purchase which products.

the positive externality that its application provides to Firm M. Both total industry profits and consumer surplus are lower than under integrated monopoly, illustrating the social consequences of Firm X's inability to internalize the complementarity.

Application duopoly When Firm M monopolizes the base system market but competes with Firm X in the application market (with $\omega = 1$), both firms set a price of zero for the application, while Firm M sets a price of $p_M^b = 0.816$ for the base system. The resulting per-period profits are $\pi_M(MX, 1) = 0.544$ and $\pi_X(MX, 1) = 0$. Consumer choices are displayed in Figure 3D. Firm M trivially bundles its application with the base system, forcing Firm X to sell its application at a zero price. Ordinarily, no firm would enter a pre-existing market with undifferentiated Bertrand competition if it would incur a positive entry cost. But Firm M achieves profits nearly as high as if it enjoyed an integrated monopoly, because by selling its application at a zero price it increases the demand for its base system. As with integrated monopoly, both total industry profits and consumer surplus are higher than under bilateral monopoly, so from an ex post perspective Firm M's entry into the application market is both privately and socially desirable if its investment costs are not too high.

4.3 Dynamic equilibrium

Though the game has an infinite horizon, it can be solved by backward induction from the absorbing states. Proposition 2 applies the results of the basic framework to this model.

Proposition 2. *In the Base System model, if the prior distribution over demand is sufficiently pessimistic ($\gamma < 0.679$) then the region of forgone invention has positive measure.*

To gain a more economic interpretation of this result, it is helpful to look a little more closely at the three effects that drive Firm M's investment incentives. When demand for the application is high, Firm M chooses to invest ex post if the gain from investing more than offsets the fixed investment cost:

$$c_M < \frac{1}{1-\delta}\big(\pi_M(MX, 1) - \pi_M(X, 1)\big). \tag{5}$$

In contrast, if Firm X were unable to invest this then Firm M would enter ex ante if

$$c_M < \frac{\gamma}{1-\delta}\big(\pi_M(M, 1) - \pi_M(\emptyset)\big) = \hat{c}_M. \tag{6}$$

Most of the difference between Eq. 5 and Eq. 6 is due to two effects: First, if Firm X has already revealed high demand for the application, Firm M receives a higher payoff even if it does not invest because the availability of Firm X's application makes Firm M's base system

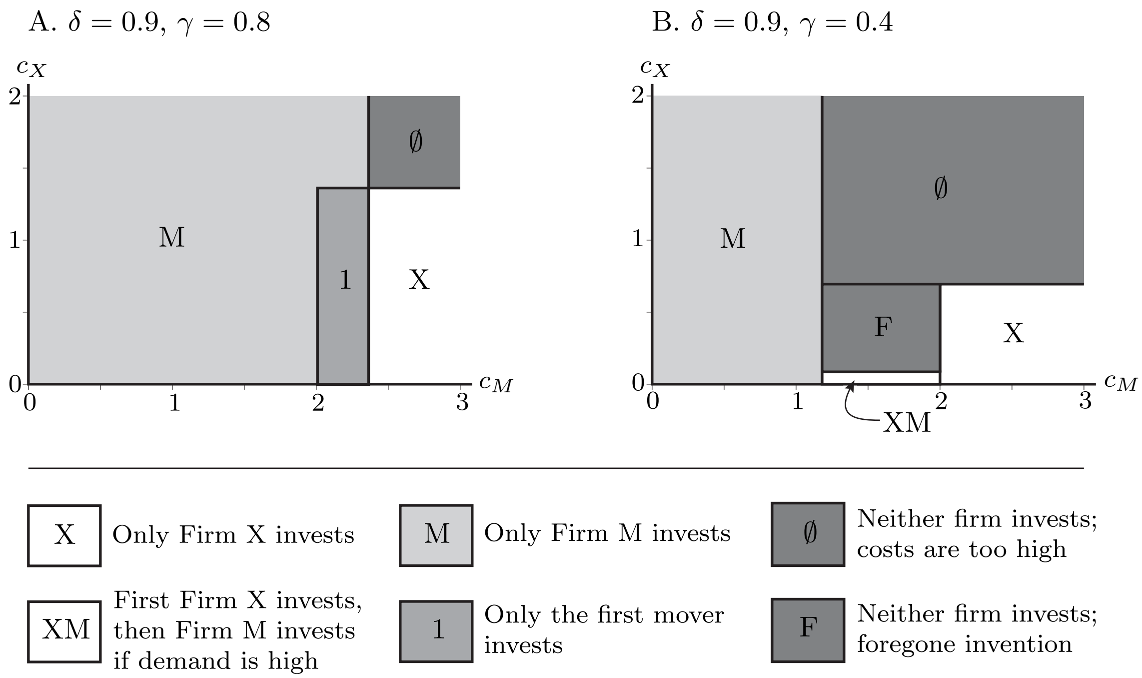

FIGURE 4: EQUILIBRIUM OUTCOME AS A FUNCTION OF COSTS IN THE BASE SYSTEM MODEL Each graph displays the cost space, with c_M on the horizontal axis and c_X on the vertical axis. Regions are shaded according to which firm invests first, and labeled according to the type of equilibrium outcome.

more valuable; i.e., $\pi_M(X, 1) > \pi_M(\emptyset)$. This *complementarity effect* reduces the incentive for Firm M to invest. Second, if Firm X has already invested and demand is high, then Firm M's expected payoff from investing is higher, increasing its incentive to invest. This *demand revelation effect* is reflected in the absence of γ from Firm M's ex post entry rule (Eq. 5). A third effect, the *competitive effect*, stems from competition in the application market, but it exerts almost no influence on Firm M's decision because Firm M can achieve nearly as much profit by trivial bundling against Firm X as it can by monopolizing both markets. The demand revelation effect swamps the competitive effect except when γ is extremely close to 1, and outweighs the complementarity effect as well when $\gamma < 0.679$.

To graph the equilibrium outcome, let $\delta = 0.9$, and consider two cases, $\gamma = 0.8$ and $\gamma = 0.4$. When $\gamma = 0.8$, equilibria in which Firm X and Firm M invest ex ante share a region of overlap (labeled as the region of first mover advantage in in Figure 4A).[23] Since $\gamma = 0.8 > 0.679$, there is no region of foregone invention.

[23]In parts of this region, either Firm M or Firm X, but not both, could invest in pure strategy equilibrium. In these regions there is also an unstable mixed strategy equilibrium, but it is not robust to a small first-mover advantage for either firm.

When $\gamma = 0.4$, the regions in which Firm X and Firm M may invest ex ante do not overlap, and the region of foregone invention fills the gap: in equilibrium, neither firm invests even in situations in which investment could be privately profitable under non-equilibrium strategies. This is illustrated in Figure 4B. For cost vectors in the region of foregone invention, the threat of ex post entry by Firm M deters Firm X from investing ex ante.

The region of foregone invention reaches its maximum area at an intermediate level of uncertainty, $\gamma = 0.340$. This occurs for two reasons: First, if demand is less likely to be high, then entry is relatively unlikely even if foregone invention were not a problem. Second, if demand is more likely to be high (but still $\gamma < 0.679$), then ex post incentives are more in line with ex ante incentives. Additionally, for any fixed $\gamma \in (0, 0.679)$ the region of foregone invention grows with increasing δ. Although greater patience gives firms a greater incentive to invent new products, it does not help align ex ante incentives with ex post incentives, so the range of entry costs over which invention is foregone increases.

4.4 Strategies to encourage invention

The region of foregone invention is troubling, both for society and for Firm M: opportunities to invent new products may go unexploited even though they offer potential for private profit and expansion of Firm M's market. This section considers several policy interventions, both private and public. However, none of them solves the problem without negative side effects.

4.4.1 Private strategies for encouraging invention

Mergers From both a social standpoint and from the firms' perspective, it is usually preferable for whichever firm has lower costs to invent the application, and then for Firm M to market it, rather than for Firm M to enter ex post. First, suppose that Firm M can offer to buy Firm X ex post. If Firm M's entry costs are relatively low, it can credibly threaten to enter the application market on its own, giving Firm X an outside option of $\frac{1}{1-\delta}\pi_X(MX, 1) = 0$. Firm M's influence over Firm X's outside option leads to a low buyout price, and correspondingly low ex ante incentives for Firm X to enter. If instead Firm M's entry costs too high for it to enter ex post, it offers Firm X a buyout price at least as high as Firm X's outside option, $\frac{1}{1-\delta}\pi_X(X, 1) = \frac{1}{1-\delta}0.172$. Thus the prospect of merging ex post actually increases the size of region of foregone invention, because merging offers Firm X greater incentives to enter when Firm M poses no ex post threat.

If the firms could merge ex ante, then they could jointly internalize the complementarity and eradicate the problem of foregone invention. Merging ex ante could be problematic, however. First, if entry costs were private information then Firm M would face an adverse selection problem in identifying which firms to merge with. Second, even if the correct Firm X could be identified, the transaction costs of the merger might outweigh the difference in entry costs between the two independent firms. Such transaction costs could include not only the accounting costs of the merger, but also the effect on the incentives of the software engineers of moving from a small firm to a larger firm.

Collusion Firm M and Firm X, having both introduced competing applications, could collude to keep the prices of their applications above zero. Joint profit maximization implies that $p^b_M = 0.667$ and $p^a_M = p^a_X = 0.167$, yielding profits per period of 0.495 for Firm M and 0.051 for Firm X if they split the application market equally. However, Firm M can actually earn higher per period profits ($\pi_M(MX, 1) = 0.544$) by driving the application price to zero. Collusion on positive application prices is possible only if Firm X restricts its output to much less than half of the total number of applications, leaving Firm X with little inducement to innovate.

Building a reputation for enabling complements The Base System model considers a single application, but if a new potential application market arises in each period, we might expect there to be an equilibrium in which Firm M does not enter certain already-occupied complementary markets, because the independent firms will cease innovating if they ever observe Firm M enter a forbidden complementary market. Analysis of such a model is beyond the scope of this paper, but Gawer and Cusumano (2002) theorize that Intel has pursued the strategy of attempting to coordinate on such an equilibrium, characterizing its strategy as "communicating commitment to third parties."[24]

4.4.2 Public policies for inducing invention

Patent protection A simple patent policy can eliminate the appropriability problem facing Firm X. Unfortunately, since patents prevent competition, they can reduce social surplus whenever they are not necessary to induce investment. This phenomenon manifests in at least four specific ways: First, the costs of reduced competition are evident in the Base System model whenever Firm M's subsequent entry does not deter Firm X from innovating ex ante. In these situations, it is socially desirable ex ante for Firm M to enter a market occupied by Firm X. Second, though the Base System model does not allow for vertical differentiation,

[24] Allowing for a new application each period would require a dynamic game with an infinite state space, for which there is no folk theorem to invoke and for which static prices and profits are difficult to compute. It can be shown that if total expenditure in the markets for applications and base systems is bounded, then Firm M cannot commit to encouraging complements even if the number of applications is unbounded. If instead the lifetimes of applications are uniformly bounded, then the folk theorem of Dutta (1995) applies. But an "application" in the model is really an application category, which may have an indefinite lifetime.

a lack of competition can be particularly problematic in markets in which incremental quality improvement is important because competition can induce firms to invest more in improving quality. Heeb (2003) shows that Firm M is likely the stronger incremental innovator in such a situation, so long as Firm X is not induced to exit. Third, the Base System model does not allow for horizontal differentiation, but in markets in which consumers have varying horizontal tastes it may be beneficial to have multiple vendors to satisfy different types of consumers. Fourth, there may be many markets that Firm M will not enter, and in those markets it may be beneficial to allow entry by other independent firms who do not hold ex post advantages. Whether a patent policy is socially beneficial in any given market is subject to the balance between these effects and the social benefit of rewarding innovators.[25]

In the case of computer software, the issue of patent protection is often moot. While entire software applications are generally not patentable, many applications make use of patented software technologies. In particular, base system vendors like Microsoft and Intel tend to hold many of these sorts of patents—enough to potentially head off attempts by complementors to patent application categories.[26]

Mandatory divestiture In the model, requiring Firm M to divest itself of any applications that it develops makes little sense: the spun-off firm (call it Firm Y) would drive the application price to zero. However, either vertical or horizontal product differentiation would allow both Firm X and Firm Y to earn modest profits when they compete. Not only would this offer greater incentives for Firm X to innovate ex ante, it would also give Firm X a greater opportunity to enter a market already occupied by Firm Y to provide a horizontally differentiated alternative.[27] Vertical product differentiation combined with horizontal differentiation would give both firms incentives for incremental quality improvement.[28] However, mandatory divestiture cannot eliminate the problem of foregone invention: Firm M still has some ex post incentive to develop the application so that competition between Firms X and Y will drive down application prices. Another drawback is that Firm M's static pricing policy no longer internalizes the complementarity between the products. Whether mandatory divestiture would be desirable depends on the balance among the social benefits of invention, incremental improvement, product variety, and static efficiency.

[25]More desirable would be a finely tuned patent system that awards a patent only when necessary to induce invention, but the information requirements of such a system are heavy: it would need to know the prior distribution of demand as well as the cost structures of all potential entrants.

[26]See Bessen (2005) for a closer examination of this idea.

[27]Whether such entry is desirable depends on the specification of demand and entry costs.

[28]Heeb (2003) shows that Firm Y would be a stronger incremental innovator if it were integrated with Firm M, as long as Firm X is not induced to exit. On the other hand, Firm X is more likely to be induced to exit when facing an integrated competitor than when facing a symmetric competitor.

Regulation In recent years, antitrust authorities in the E.U. and U.S. have attempted to prevent Microsoft from excluding rivals by means other than pricing. A more severe option is to regulate Firm M's pricing so that, when it enters an already-occupied market, it cannot undercut its rival or bundle its application with any other products; this would restore some of Firm X's ex post profits. Because it prevents bundling, it also reduces the incentive for Firm M to enter ex post. This alleviates, but does not fully solve the foregone invention problem. Outside the model, such a policy would give Firm M an incentive to compete on quality because it could not compete on price. On the down side, such a policy invites possibly inefficient ex post entry by low-cost third parties. It also begs the question of how to define the boundaries between application markets.

5 Additional examples

This section examines three additional examples, each motivated by a stylized scenario in the computer software industry, and each leading to foregone invention in a qualitatively different way. All satisfy Assumptions 1–4 when viewed in the basic framework.

5.1 Bundling unrelated products

In the Base System model, Firm M could leverage its market power because it controlled the base system on which the application depended. This example instead considers two unrelated products. Firm M's ability to commit to bundling creates an artificial relationship where one does not arise naturally, allowing it to extend its market power to new markets.

This example is motivated in part by Microsoft's dominance in the market for business productivity software, and its practice of bundling separate applications together under the name "Microsoft Office." Nalebuff (2004), Carlton and Waldman (2002), and Bakos and Brynjolfsson (2000) show that bundling can serve as an entry deterrent, and Bakos and Brynjolfsson speculate that innovation incentives in an empty market might be distorted by the presence of a potential bundler. Here, I show that Microsoft's ex post option to bundle future products into Office may deter the ex ante invention of these products. In the works cited above, the monopolist initially monopolizes multiple markets and tries to prevent entry into one of them. Here, the incumbent monopolizes only one market, while another market lies unoccupied, with unknown demand.

The demand system is identical to Section 4.1, except that consumers can obtain value from product a without product b. In each static market, Firm M first chooses which bundles to offer from among the products it has developed, and then both firms set prices simultaneously. "Mixed" bundling (in which a firm offers a bundle alongside independent products) is allowed. Proposition 3 shows that foregone invention can arise in this model.

Proposition 3. *In the Bundling model, if the prior distribution over demand is sufficiently pessimistic ($\gamma < 0.398$) then the region of foregone invention has positive measure.*

Commitment is necessary for invention to be foregone. When $s = MX$, Firm M would want to undercut Firm X's price in the application market if it could renege on its bundling commitment, so the price of product a would fall to zero under Bertrand competition. Thus Firm M would not enter the market for product a ex post if it could not commit to bundling. This suggests a simple public policy remedy: do not allow Firm M to bundle any unrelated products together with a product over which it has market power.

In this example, the two products are completely unrelated. But the results carry over to the case of partial complementarity. If the new product a were complementary to Firm M's existing product b, then Firm M might enter the market for product a with a bundle to extract more profit from consumers. Firm X is able to earn positive profits if Firm M enters with bundling, but not if Firm M enters without bundling. Thus when complementarity is strong enough to induce Firm M to enter even if it could not commit to bundling, its ability to commit to bundling has a beneficial effect on Firm X's ex ante invention incentives. Hence a blanket ban on bundling may not always be beneficial.

5.2 Product features

This example considers two firms with horizontally differentiated products, with the potential to add a new feature. Neither firm is privileged in the ex post market; instead, that the new feature brings them into closer competition leads to an excessive incentive to invest ex post. The model illustrates that foregone invention may occur even in a symmetric situation, and shows that market power need not take the form of monopoly in order to pose a deterrent to innovation in related markets.

In the world of software, one example of such a pair of horizontally differentiated products might be a visual web page editor and an HTML code editor. Though the products differ, there is an opportunity to add the ability to manipulate multimedia files to either or both of them. The demand for this feature, though unknown, is orthogonal to the horizontal differentiation between the existing products. If only one of the two products offers the new feature, some customers may switch to the one with the new feature.

The example is constructed as follows. Consumer i's valuation of Firm M's product is θ_i^0, while her valuation of Firm X's product is $1 - \theta_i^0$. Consumer valuations θ_i^0 are distributed uniformly on $[0, 1]$. This horizontal differentiation is calibrated so that, prior to the introduction of any new features, all consumers are served in equilibrium and each firm earns monopoly profits. Consumer i's valuation of the new feature, θ_i^1, with probability γ is distributed uniformly on $[0, 1]$, and is 0 for all consumers otherwise. Proposition 4 shows that firms may be deterred from adding new features to their products by the possibility that doing so will bring them into closer competition with currently distant competitors.

Proposition 4. *In the Product Features model, if the prior distribution over demand is sufficiently pessimistic ($\gamma < 0.707$) then the region of foregone invention has positive measure.*

The ex post incentive to develop the new feature is strong, since a firm will suffer badly if only its competitor offers the new feature. Thus it is the prospect of losing market power, rather than gaining market power, that drives ex post entry and reduces ex ante investment.

5.3 Divided Technical Leadership

This section presents a situation in which Firm M's profits in a related market are so threatened by Firm X's introduction of a new product that Firm M is induced to develop a new competing product ex post in order to protect its existing business. The model exemplifies some aspects of the "divided technical leadership" (DTL) hypothesis of Bresnahan and Greenstein (1996). Carlton and Waldman (2002) also examines the closely related question of how a monopolist can extend its monopoly to the market for a newly emerging substitute. My purpose is not to construct the definitive model of DTL, but rather to propose that DTL suggests that platform leaders may seek to discourage the invention of complementary platform layers. The example also illustrates how the conditions for foregone invention are qualitatively different when a dominant firm perceives its competitors' investments as strategic complements.

The scenario is a stylized conception of Microsoft's entry into the internet browser market once Netscape's browser became popular. Microsoft feared that the combination of Netscape's browser and Sun Microsystems' Java Virtual Machine (JVM), as a middleware layer on top of Windows, could potentially "commoditize" the underlying operating system.[29] If Netscape and Sun succeeded, Microsoft might be left to compete on price against other operating system vendors while Netscape and Sun assumed the mantle of platform leadership. The DTL hypothesis claims that such transitions are the usual course of change in the computer industry: a new entrant develops a complementary computing layer to serve a market segment distinct from that of the current platform leader, but once it achieves success in its own market it exploits changing trends in the industry to wrest platform leadership from the incumbent. The model suggests that, before Netscape popularized the browser, Microsoft faced only moderate incentives to enter the browser market because it could gain at most monopoly profits in that market. Microsoft judged, perhaps rightly, that it could better allocate its development resources to other projects ex ante. But once the browser-JVM combination threatened to become successful, Microsoft entered the browser

[29]See Bresnahan (2001) for a discussion of Microsoft's motivations based on internal strategy documents.

market in order to counter the challenge to its leadership role. From Microsoft's perspective, investments were strategic complements: its incentives to invest were greater once Netscape and Sun also invested, because if the Netscape-Sun middleware product had succeeded then Microsoft could have faced severe consequences in its existing markets.

In this model, Firm X can invent a complementary application (product a), and if it turns out to be popular then Firm X can invent a rival base system (product d) two periods later at cost c_d in the second period. Firm X's application works with either base system. Firm M can also develop the application, but Firm M's application works only with Firm M's base system; Firm M cannot invent a new base system. The only uncertainty is over the demand for the application, which is the same as in the Base System model. I assume that each consumer buys only one application when Firm M is the only base system vendor (in which case the cross-platform compatibility of Firm X's application's offers no advantage), so if Firm M bundles its application then none of Firm M's customers buy Firm X's application. Consumers' values for Firm M's base system, θ_i^b, are distributed uniformly on $[1, 2]$ and each consumer's value for Firm X's base system is $3 - \theta_i^b$.[30]

Proposition 5. *In the DTL model, if* $c_d < \frac{1}{1-\delta} 0.5$ *and* $\delta > 0.113$ *then the region of foregone invention has positive measure.*

In contrast to the previous examples, foregone invention in the DTL example does not involve the level of uncertainty, γ. This is because Firm M's ex post incentive to prevent entry into its base system market exceeds its ex ante incentive to invent a complementary application of its own, even if $\gamma = 1$. Indeed, if Firm X were to develop a rival base system, Firm M's profits would be cut roughly in half. In contrast, when Firm X does not enter, Firm M has little interest in the application market because even if demand is sure to be high it can increase its profits by only about 30%. Hence Firm M perceives entry as strategic complements, even while Firm X perceives entry as strategic substitutes. In such a situation, uncertainty is not required to put a wedge between Firm M's ex ante and ex post incentives, and hence Firm X can be deterred from entering even when demand is common knowledge.[31]

[30] Consumers' base system valuations are higher by 1 than in the product features model, so that under symmetric conditions Firm X's new base system captures some of Firm M's potential base system customers.

[31] In reality, Netscape did develop a browser and Sun did develop its JVM. In the context of the model, this could mean that their development costs were sufficiently low that Microsoft's ability to force the browser price to zero and to exclude them from the base system market was not sufficient to deter their initial investment. Outside the model, it is also possible that Netscape and Sun misjudged Microsoft's ability to overcome the network externalities that they had built up through wide distribution of the browser and JVM. On the other hand, Netscape and Sun may have judged the situation properly and merely experienced a bad realization from among a range of possible outcomes.

6 Discussion

This paper deals with the dynamic consequences of ex post competition when heterogeneous inventors face uncertain demand. Uncertainty leads only low–entry cost firms to invent ex ante, but high–entry cost firms may wish to enter ex post if demand is revealed to be high. The prospect of ex post competition from high–entry cost firms dampens the ex ante incentives of low–entry cost firms, so that invention is foregone over an intermediate region of entry cost profiles. For the region of foregone invention to have positive measure it is sufficient that at least one firm perceive investments as strategic substitutes, and that with positive probability demand is high enough that the other firm enters ex post when its entry costs are high enough that it would not enter ex ante. Although foregone invention can occur in symmetric situations, the problem is more likely to occur when one firm earns high monopoly profits and low duopoly profits, while the other firm earns high duopoly profits.

Throughout, I have taken an agnostic stance regarding the entry costs of each potential inventor, expressing the results either as contingent on a particular cost profile or as statements about regions of cost profiles. The underlying idea is that the development of any particular invention poses a unique set of challenges, and each potential inventor will approach these challenges in its own way. In the context of the Base System model, where Firm M is interpreted as Microsoft, for one kind of application one may believe that Microsoft has a low entry cost due to its knowledge of the inner workings of the Windows operating system. For another application, one may believe that a startup funded by venture capital has a low entry cost, due to the strong incentives that its financial structure provides to its software engineers. These kinds of beliefs, aggregated across possible inventions, could be interpreted as a probability distribution over cost profiles, which in principle could be combined with knowledge about the region of foregone invention to yield the proportion of inventions expected to be foregone. Empirically, such aggregation is hardly feasible, but the thought experiment yields a qualitative perspective on foregone invention.

As for Microsoft, the results in this paper suggest that its monopoly over operating systems may pose a deterrent to innovation that has the potential to harm both society and Microsoft, but also that many policies that might eliminate or reduce this threat could have other negative effects on innovation. Perhaps it is helpful to recognize that Microsoft is merely a dominant firm in the operating system market, rather than a true monopolist. The availability of multiple operating systems can help independent software developers in the ex post market, because they can market their application software for several platforms, while each operating system vendor can trivially bundle only with its own operating system. Competition can also drive incremental improvements in operating systems. Of course, if there are benefits of standardization on Windows then the continued presence of alternative operating systems means that some of these benefits are foregone. Likewise, development costs may be higher for application software that runs on multiple operating systems.

Appendix: Proofs

Lemma 1. *Let $s^*_j(\omega, c_j)$ and $z^*_j(\omega, c_j)$ be the market structure and Firm j's investment cost, respectively, after Firm j's sequentially optimal choice after Firm $-j$ has entered and revealed ω; i.e., $s^*_j(\omega, c_j) = MX$ and $z^*_j(\omega, c_j) = c_j$ if $\pi^2_j(MX, \omega) - c_j > \pi^2_j(-j, \omega)$, while $s^*_j(\omega, c_j) = -j$ and $z^*_j(\omega, c_j) = 0$ otherwise.*[32] *There exists a subgame perfect equilibrium in which neither firm enters the market if and only if*

$$\mathbb{E}\big[\pi^1_j(j, \omega) + \pi^2_j\big(s^*_{-j}(\omega, c_{-j}), \omega\big)\big] - c_j \leq \pi^1_j(\emptyset) + \pi^2_j(\emptyset) \tag{7}$$

for both j. Furthermore, this is the unique subgame perfect outcome if and only if

$$\mathbb{E}\big[\pi^1_j(MX, \omega) + \pi^2_j(MX, \omega)\big] - c_j < \mathbb{E}\big[\pi^1_j(-j, \omega) + \pi^2_j\big(s^*_j(\omega, c_j), \omega\big) - z^*_j(\omega, c_j)\big], \tag{8}$$

for some firm j, and also the inequality in Eq. 7 is strict for firm $-j$.

Proof. Backward induction reduces the game to the following normal form in the first period:

	in^1_X	out^1_X
in^1_M	$\mathbb{E}\big[\pi^1_M(MX, \omega) + \pi^2_M(MX, \omega)\big] - c_M$, $\mathbb{E}\big[\pi^1_X(MX, \omega) + \pi^2_X(MX, \omega)\big] - c_X$	$\mathbb{E}\big[\pi^1_M(M, \omega) + \pi^2_M(s^*_X(\omega, c_X), \omega)\big] - c_M$, $\mathbb{E}\big[\pi^1_X(M, \omega) + \pi^2_X(s^*_X(\omega, c_X), \omega) - z^*_X(\omega)\big]$
out^1_M	$\mathbb{E}\big[\pi^1_M(X, \omega) + \pi^2_M(s^*_M(\omega, c_M), \omega) - z^*_M(\omega)\big]$, $\mathbb{E}\big[\pi^1_X(X, \omega) + \pi^2_X(s^*_M(\omega, c_M), \omega)\big] - c_X$	$\pi^1_M(\emptyset) + \pi^2_M(\emptyset)$, $\pi^1_X(\emptyset) + \pi^2_X(\emptyset)$

Then Eq. 7 (for both firms) holds if and only if $(\text{out}^1_M, \text{out}^1_X)$ is a Nash equilibrium of this normal form. Also, Eq. 7 with strict inequality for firm $-j$ and weak inequality for firm j, combined with Eq. 8 for firm j, are necessary and sufficient for there to be no other Nash equilibria. ■

Lemma 2. *Under Assumptions 1–4, let $\tilde{c}_M \equiv \sup\{c_M : \Pr[s^*_M(\omega, c_M) = MX] > 0\}$. Suppose that investments are strategic substitutes for Firm X; then there exists $\varepsilon > 0$ such that, given entry cost $\hat{c}_X - \varepsilon$, (i) if $c_M < \tilde{c}_M$ then Firm X does not enter the empty market in the first period in any subgame perfect equilibrium, and (ii) if $c_M > \tilde{c}_M$ then Firm X enters the empty market in the first period in every subgame perfect equilibrium.*

Proof. Positive profits and common demand imply that $\tilde{c}_M > 0$. This fact combined with common demand and competitiveness implies that $\mathbb{E}\big[\pi^2_X(X, \omega)\big] > \mathbb{E}\big[\pi^2_X(s^*_M(\omega, c_M), \omega)\big]$ for all $c_M < \tilde{c}_M$. Hence, given an entry cost profile $(c_M, \hat{c}_X)$ with $c_M < \tilde{c}_M$, Firm X's strict best response (in the first

[32] The assumption that a firm indifferent to entry will not enter plays no role in the analysis since each firm strictly prefers to either enter or not enter for almost all cost vectors.

period reduced game) to out_M^1 is out_X^1:

$$\begin{aligned}\hat{c}_X &\equiv \mathbb{E}\big[\pi_X^1(X,\omega)+\pi_X^2(X,\omega)\big]-\pi_X^1(\emptyset)-\pi_X^2(\emptyset)\\ &> \mathbb{E}\big[\pi_X^1(X,\omega)+\pi_X^2(s_M^*(\omega,c_M),\omega)\big]-\pi_X^1(\emptyset)-\pi_X^2(\emptyset).\end{aligned} \tag{9}$$

That investments are strategic substitutes for Firm X further implies that Firm X's strict best response (in the first period reduced game) to in_M^1 is out_X^1:

$$\begin{aligned}\hat{c}_X &= \mathbb{E}\big[\pi_X^1(X,\omega)+\pi_X^2(X,\omega)\big]-\pi_X^1(\emptyset)-\pi_X^2(\emptyset)\\ &> \mathbb{E}\big[\pi_X^1(MX,\omega)+\pi_X^2(MX,\omega)-\pi_X^1(M,\omega)-\pi_X^2(M,\omega)\big]\\ &\geq \mathbb{E}\big[\pi_X^1(MX,\omega)+\pi_X^2(MX,\omega)-\pi_X^1(M,\omega)-\big(\pi_X^2(s_X^*(\omega,c_X),\omega)-z_X^*(\omega,c_X)\big)\big],\end{aligned} \tag{10}$$

where the strict inequality is by strategic substitutes and the weak inequality is because Firm X will enter ex post only if by doing so it improves its profits. Choose $\varepsilon>0$ sufficiently small that both of these strict best responses continue to hold at $c_X=\hat{c}_X-\varepsilon$. Then, given $(c_M,\hat{c}_X-\varepsilon)$ with $c_M>\tilde{c}_M$, Firm X's strict best response is in_X^1. The argument for (ii) is similar. ■

Proof of Proposition 1 (page 9). Eq. 4 and competitiveness imply that $\hat{c}_M<\tilde{c}_M$, so by Lemma 2, given entry cost profile $(\hat{c}_M,\hat{c}_X-\varepsilon_X)$ with $\varepsilon_X>0$ sufficiently small, Firm X does not enter in any subgame perfect equilibrium even though it would strictly prefer to enter given an entry cost profile $(c_M,\hat{c}_X-\varepsilon_X)$ with $c_M>\tilde{c}_M$. This is also true at $(\hat{c}_M+\varepsilon_M,\hat{c}_X-\varepsilon_X)$, for $\varepsilon_M>0$ sufficiently small, where Firm M also does not enter in any subgame perfect equilibrium. ■

Proof of Proposition 2 (page 15). First, $\hat{c}_M=\frac{\gamma}{1-\delta}\big(0.546-0.25\big)=\frac{\gamma}{1-\delta}0.296$. Second, $s_M^*(1,\hat{c}_M)=MX$ if $c_M<\frac{1}{1-\delta}\big(0.544-0.343\big)=\frac{1}{1-\delta}0.201$ and $s_M^*(1,\hat{c}_M)=X$ otherwise, while $s_M^*(0,\hat{c}_M)=X$. Third, by assumption $\hat{c}_M<\frac{1}{1-\delta}0.201$, so $\mathbb{E}\big[\pi_X^2(X,\omega)\big]=\frac{\gamma\delta}{1-\delta}0.172>\mathbb{E}\big[\pi_X^2(s_M^*(\omega,\hat{c}_M),\omega)\big]=0$. Finally, Firm X perceives investments as strategic substitutes because Firm X always earns zero profits when it does not invest. Thus Proposition 1 applies. ■

Proof of Proposition 3 (page 21). Derivation of the following static bundles, prices, and profits is straightforward and thus omitted. When $s=\emptyset$ or $\omega=0$, $p_M^b(\emptyset)=0.5$ and $\pi_M(\emptyset)=0.25$. When $s=M$ and $\omega=1$, Firm M offers a bundle at $p_M^{ba}(M)=0.862$ as well as each product independently at $p_M^b(M)=p_M^a(M)=0.667$; its profits are $\pi_M(M,1)=0.549$. When $s=X$ and $\omega=1$, $p_M^b(X)=p_X^a(X)=0.5$ and $\pi_M(X,1)=\pi_X(X,1)=0.25$. When $s=MX$ and $\omega=1$, Firm M offers a bundle at $p_M^{ba}(MX)=0.607$, while Firm X offers product a at $p_X^a(MX)=0.245$. Profits are $\pi_M(MX,1)=0.369$ and $\pi_X(MX,1)=0.067$. Observe that $\hat{c}_M=\frac{\gamma}{1-\delta}\big(0.549-0.25\big)=\frac{\gamma}{1-\delta}0.299$. Second, $s_M^*(1,c_M)=MX$ if $c_M<\frac{1}{1-\delta}\big(0.369-0.25\big)=\frac{1}{1-\delta}0.119$ and $s_M^*(1,c_M)=X$ otherwise, while $s_M^*(0,c_M)=X$. Third, by assumption $\hat{c}_M<\frac{1}{1-\delta}0.119$, so $\mathbb{E}\big[\pi_X^2(X,\omega)\big]=\frac{\gamma\delta}{1-\delta}0.25>\mathbb{E}\big[\pi_X^2(s_M^*(\omega,\hat{c}_M),\omega)\big]=\frac{\gamma\delta}{1-\delta}0.067$. Finally, Firm X perceives investments as strategic substitutes because Firm X earns zero profits whenever it does not invest. Thus Proposition 1 applies. ■

Proof of Proposition 4 (page 22). Derivation of the following static prices and profits is straightforward and thus omitted. When $s=\emptyset$ or $\omega=0$, each $p_j(\emptyset)=0.5$ and $\pi_j(\emptyset)=0.25$ for $j=M,X$. When $s=M$ and $\omega=1$, $p_M(M)=0.832$, $p_X(M)=0.534$, $\pi_M(M,1)=0.472$, and $\pi_X(M,1) = 0.195$. When $s = X$ and $\omega = 1$, $p_X(X) = 0.832$, $p_M(X) = 0.534$, $\pi_X(X,1) = 0.472$, and $\pi_M(X,1) = 0.195$. When $s = MX$ and $\omega = 1$, $p_M(MX) = p_X(MX) = 0.75$ and $\pi_M(MX,1) = \pi_X(MX,1) = 0.352$. Observe that $\hat{c}_M = \frac{\gamma}{1-\delta}\big(0.472 - 0.25\big) = \frac{\gamma}{1-\delta}\,0.222$. Second, $s^*_M(1,c_M) = MX$ if $c_M < \frac{1}{1-\delta}\big(0.352 - 0.25\big) = \frac{1}{1-\delta}\,0.102$ and $s^*_M(1,c_M) = X$ otherwise, while $s^*_M(0,c_M) = X$. Third, by assumption $\hat{c}_M < \frac{1}{1-\delta}\,0.102$, so $\mathbb{E}\big[\pi^2_X(X,\omega)\big] = \frac{\delta}{1-\delta}\big(\gamma\,0.534 + (1-\gamma)\,0.25\big) > \mathbb{E}\big[\pi^2_X(s^*_M(\omega,\hat{c}_M),\omega)\big] = \frac{\delta}{1-\delta}\big(\gamma\,0.352 + (1-\gamma)0.25\big)$. Finally, Firm X perceives investments as strategic substitutes, since $\mathbb{E}\big[\pi^t_X(MX,\omega) - \pi^t_X(M,\omega)\big] = \gamma\big(0.352 - 0.195\big) < \mathbb{E}\big[\pi^t_X(X,\omega)\big] - \pi^t_X(\emptyset) = \gamma\big(0.472 - 0.25\big)$. Thus Proposition 1 applies. ■

Lemma 3. *When $c_d < \frac{1}{1-\delta}\,0.5$, the DTL model is equivalent to the basic framework with*

$$\pi^2_M(X,1) = \delta\,1.060 + \frac{\delta^2}{1-\delta}\begin{cases}0.431 & \text{if } c_M > \frac{\delta}{1-\delta}\,0.069\\ 0.5 & \text{if } c_M < \frac{\delta}{1-\delta}\,0.069,\end{cases} \tag{11}$$

$$\pi^2_X(X,1) = \delta\,0.238 + \frac{\delta^2}{1-\delta}\begin{cases}0.776 & \text{if } c_M > \frac{\delta}{1-\delta}\,0.069\\ 0.5 & \text{if } c_M < \frac{\delta}{1-\delta}\,0.069,\end{cases} \tag{12}$$

where $\pi^1_j(s,\omega) = \pi_j(s,\omega)$ and $\pi^2_j(s,\omega) = \frac{\delta}{1-\delta}\pi_j(s,\omega)$ for all $(s,\omega) \neq (X,1)$.

Proof. There are six relevant market structures, $s \in \{\emptyset, M, X, MX, XX, MXX\}$, where XX indicates that Firm X has developed both an application and a base system. Derivation of the following static prices and profits is straightforward and thus omitted. When $s = \emptyset$ or $\omega = 0$, $p^b_M(\emptyset) = 1$ and $\pi_M(\emptyset) = 1$. When $s = M$ and $\omega = 1$, $p^b_M(M) = 1.272$, $p^a_M(M) = 0.340$, and $\pi_M(M,1) = 1.319$. When $s = X$ and $\omega = 1$, $p^b_M(X) = 1.222$, $p^a_X(X) = 0.488$, $\pi_M(X,1) = 1.060$, and $\pi_X(X,1) = 0.238$. Firm X's application is popular. When $s = MX$ and $\omega = 1$, $p^a_M(MX) = p^a_X(MX) = 0$, $p^b_M(MX,1) = 1.549$, $\pi_M(MX,1) = 1.316$, and $\pi_X(MX,1) = 0$. No consumers purchase Firm X's application. When $s = XX$ and $\omega = 1$, $p^b_M(XX) = 0.928$, $p^d_X(XX) = p^{da}_X(XX) = 1.297$, and $p^a_X(XX) = 0.655$. Profits are $\pi_M(XX,1) = 0.431$ and $\pi_X(XX,1) = 0.776$.[33] When $s = MXX$ and $\omega = 1$, $p^a_M(MXX) = p^a_X(MXX)$, $p^b_M(MXX) = p^d_X(MXX) = 1$, and $\pi_M(MXX,1) = \pi_X(MXX,1) = 0.5$. Solve backward from the absorbing state, MXX, to states X, M, and MX. If $s = XX$, then Firm M invests if $c_M < \frac{1}{1-\delta}\,0.069$. If $s = MX$ then Firm X cannot invest, because its application is not popular. If $s = M$, then Firm X does not invest. If $s = X$ and one period has already elapsed since Firm X invented the application, then Firm X's best response to no investment by Firm M is to invest if $c_d < \frac{1}{1-\delta}\,0.538$; Firm M's mutual best response is indeed not to invest if $c_M > \frac{1}{1-\delta}\,0.069$. Firm X's best response to investment by Firm M in this situation is to invest if $c_d < \frac{1}{1-\delta}\,0.5$; Firm M's mutual best response is indeed to invest if $c_M < \frac{1}{1-\delta}\,0.069$. If $s = X$ and Firm X has just invented the application, Firm M can switch to state MX by investing, the incentives for which are accounted for in the basic framework. ■

Lemma 4. *Assuming that investments are strategic substitutes for Firm X, suppose that second*

[33] There is actually a superior, but quantitatively insignificant solution in which Firm X also offers its base system sans application at a slightly lower price. The change in profits for each firm is less than 10^{-4}.

period profits are much more important than first period profits and

$$\mathbb{E}\big[\pi_M^2(MX,\omega) - \pi_M^2(X,\omega)\big] > \mathbb{E}\big[\pi_M^2(M,\omega)\big] - \pi_M^2(\emptyset). \tag{13}$$

Then the region of foregone invention has positive measure.

Proof. Since $\mathbb{E}\big[\pi_M^2(\cdot,\omega)\big]$ is supermodular in investment, the measure of ω for which $\pi_M^2(MX,\omega) - \pi_M^2(X,\omega) > \mathbb{E}[\pi_M^2(M,\omega)] - \pi_M^2(\emptyset)$ is positive, and since first period profits are small, the measure of ω for which $\pi_M^2(MX,\omega) - \pi_M^2(X,\omega) > \mathbb{E}\big[\pi_M^1(M,\omega) + \pi_M^2(M,\omega)\big] - \big(\pi_M^1(\emptyset) + \pi_M^2(\emptyset)\big)$ is also positive. Thus Firm M enters ex post with positive probability given $\hat{c}_M$:

$$\begin{aligned} 0 &< \Pr\big[\pi_M^2(MX,\omega) - \pi_M^2(X,\omega) > \mathbb{E}\big[\pi_M^1(M,\omega) + \pi_M^2(M,\omega)\big] - \big(\pi_M^1(\emptyset) + \pi_M^2(\emptyset)\big)\big] \\ &= \Pr\big[\pi_M^2(MX,\omega) - \hat{c}_M > \pi_M^2(X,\omega)\big] = \Pr\big[s_M^*(\omega,\hat{c}_M) = MX\big]. \end{aligned} \tag{14}$$

This implies the condition in Proposition 1. ■

Proof of Proposition 5 (page 23). Suppose that $c_d < \frac{1}{1-\delta}\,0.5$. If $c_M > \frac{1}{1-\delta}\,0.069$ then satisfying Eq. 13 requires $\frac{\gamma\delta}{1-\delta}\,1.316 - \gamma\big(\delta\,1.060 + \frac{\delta^2}{1-\delta}\,0.431\big) > \frac{\gamma\delta}{1-\delta}\big(1.319 - 1\big)$, while if $c_M < \frac{1}{1-\delta}\,0.069$ then this requires $\frac{\gamma\delta}{1-\delta}\,1.316 - \gamma\left(\delta\,1.060 + \frac{\delta^2}{1-\delta}0.5\right) > \frac{\gamma\delta}{1-\delta}\big(1.319 - 1\big)$. Both these conditions are satisfied when $\delta >\ 0.113$, so by Lemma 4 invention is foregone. ■

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